

# Differential Equations

## Review for Chapter 2

### Separable Equations

Find the general solutions of the differential equations.

$$(1) \frac{dy}{dx} + 2xy = 0$$

$$(2) \frac{dy}{dx} = y \sin x$$

$$(3) 2\sqrt{x} \frac{dy}{dx} = \sqrt{1-y^2}$$

$$(4) \frac{dy}{dx} = (64xy)^{1/3}$$

$$(5) (1-x^2) \frac{dy}{dx} = 2y$$

$$(6) y^3 \frac{dy}{dx} = (y^4 + 1) \cos x$$

$$(7) \frac{dy}{dx} = \frac{(x-1)y^5}{x^2(2y^3-y)}$$

$$(8) y' = 1 + x + y + xy$$

Find the explicit particular solutions of the initial value problems

$$(9) \frac{dy}{dx} = ye^x \quad y(0) = 2e$$

$$(10) 2y \frac{dy}{dx} = \frac{x}{\sqrt{x^2-16}} \quad y(5) = 2$$

$$(11) x \frac{dy}{dx} - y = 2x^2y \quad y(1) = 1$$

$$(12) \frac{dy}{dx} = 6e^{2x-y} \quad y(0) = 0$$

## Linear First Order

Find the general solution of each differential equation. If an initial condition is given, find the corresponding particular solution.

$$(13) \quad y' + 3y = 2xe^{-3x}$$

$$(14) \quad xy' + 2y = 3x \quad y(1) = 5$$

$$(15) \quad xy' + y = 3xy \quad y(1) = 0$$

$$(16) \quad (1+x)y' + y = \cos x \quad y(0) = 1$$

$$(17) \quad xy' + (2x-3)y = 4x^4$$

$$(18) \quad y' + y \cot x = \cos x$$

## Bernoulli First Order

$$(19) \quad y' = y + y^3$$

$$(20) \quad x^2 y' + 2xy = 5y^3$$

$$(21) \quad x^2 y' = xy + y^2$$

$$(22) \quad y' = (4x + y)^2$$

## Exact Equations

Verify that the given equation is exact, then solve it.

$$(23) \quad (2x+3y)dx + (3x+2y)dy = 0$$

$$(24) \quad (3x^2+2y^2)dx + (4xy+6y^2)dy = 0$$

$$(25) \quad \left(x^3 + \frac{y}{x}\right)dx + (y^2 + \ln x)dy = 0$$

$$(26) \quad \frac{dy}{dx} = \frac{-(3x^2y^3 + y^4)}{(3x^2y^2 + y^4 + 4xy^3)}$$

$$(27) \quad (\cos x + \ln y)dx + \left(\frac{x}{y} + e^y\right)dy = 0$$

## Solutions

$$(1) y = ce^{-x^2}$$

$$(2) y = ce^{-\cos x}$$

$$(3) y = \sin(c + \sqrt{x})$$

$$(4) y = (2x^{1/3} + c)^{3/2}$$

$$(5) y = \frac{c(1+x)}{1-x}$$

$$(6) \ln(y^4 + 1) = c + 4\sin x$$

$$(7) \frac{1}{3y^2} - \frac{2}{y} = \frac{1}{x} + \ln|x| + c$$

$$(8) \ln|1+y| = x + \frac{1}{2}x^2 + c$$

$$(9) y = 2e^{e^x}$$

$$(10) y^2 = 1 + \sqrt{x^2 - 16}$$

$$(11) \ln y = x^2 - 1 + \ln x$$

$$(12) y = \ln|3e^{2x} - 2|$$

$$(13) y = e^{-3x}(x^2 + c)$$

$$(14) y = x + 4x^{-2}$$

$$(15) y = 0$$

$$(16) y = \frac{1 + \sin x}{1 + x}$$

$$(17) y = x^3(2 + Ce^{-2x})$$

$$(18) y = \frac{1}{2}\sin x + c \csc x$$

$$(19) y^2 = \frac{1}{ce^{-2x} - 1}$$

$$(20) y^2 = \frac{x}{2 + cx^5}$$

$$(21) y = \frac{x}{c - \ln|x|}$$

$$(22) y = -4x + 2\tan(ax + c)$$

$$(23) x^2 + 3xy + y^2 = c$$

$$(25) 3x^4 + 4y^3 + 12y \ln x = c$$

$$(24) x^3 + 2xy^2 + 2y^3 = c$$

$$(26) 5x^3y^3 + 5xy^4 + y^5 = c$$

$$(27) \sin x + x \ln y + e^y = c$$